NORTHWESTERN UNIVERSITY | Thursday, October 6, 2005
TRAINING/SELECTION TEST

NAME: ______________________________

WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION

Problem A1. Find the sum $\displaystyle\sum_{k=0}^{n}(3k(k+1)+1)$, for $n \geq 1$.

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Problem A2. Given a fix positive integer n, find the minimum value of the following function:

$$f(x) = x^n + x^{n-2} + x^{n-4} + \cdots + \frac{1}{x^{n-4}} + \frac{1}{x^{n-2}} + \frac{1}{x^n}$$

for $x > 0$.

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Problem A3. On a large, flat field, n people ($n > 1$) are positioned so that for each person the distances to all the other people are different. Each person holds a water pistol and at a given signal fires and hits the person who is closest. When n is odd, show that there is at least one person left dry.

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Problem A4. $\mathbf{R}$ is the set of real numbers. For what $k \in \mathbf{R}$ can we find a continuous function $f : \mathbf{R} \to \mathbf{R}$ such that

$$f(f(x)) = kx^9$$

for all $x \in \mathbf{R}$.

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Problem A5. Show that for any positive integer n, there exists a positive multiple of n that contains only the digits 7 and 0.

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Problem A6. Let u_n be the number of symmetric $n \times n$-matrices whose elements are all 0's and 1's with exactly one 1 in each row. Let $u_0 = 1$. Prove that

$$u_{n+1} = u_n + n u_{n-1}$$

and

$$\sum_{n=0}^{\infty} u_n \frac{x^n}{n!} = e^{x + x^2/2}.$$